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A FURTHER NOTE ON THE AGE INDEX OF A POPULATION[1]

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In an earlier paper in these PROCEEDINGS, Pearl[2] proposed the function

$$\phi = S\left\{\frac{\Delta^2}{P}\right\}(M - M_p)$$

as a single numerical index of the age distribution of a population. In this expression S indicates summation, for all age groups given in the original statistics, of the expression in brackets; Δ is the deviation, for each age group, of the percentage of the actual population in each age group from the percentage of the same group in the standard population of Glovers[3] Life Table, denoted by P; M = mean age of living population; M_p = mean age of persons in a stationary population unaffected by migration.

In connection with certain problems now under investigation in this laboratory it becomes a matter of importance to know how sensitive this age index is to change in the size of the age groups of the original statistics. Since its original proposal the index has been used in many studies in this laboratory, and the larger experience has strengthened our confidence in its reliability as an index of significant variations in the age constitutions of populations. But it has always been used hitherto with at least 6 to 8 age groups covering the life span. Suppose the original statistics furnish only 3 age classes for the entire life span. Will this age index ϕ then give a reliable picture of the significant variations in age distribution, as we pass from city to city, or county to county?

To test this point, the obvious thing to do is to determine the correlation between the age indices for n communities, on the basis of say 3 divisions of the life span, with the age indices for the same communities on the basis of say 6 divisions of the life span. If the correlation is high it will mean

that even with the coarse grouping the index is reflecting the main differences in the form of the age distributions.

The present note reports a test of this sort. The 1915 census of Iowa[4] gives in Table I the age distribution of the inhabitants of each of the 99 counties of the state in such form as to permit the formation of the following classes:

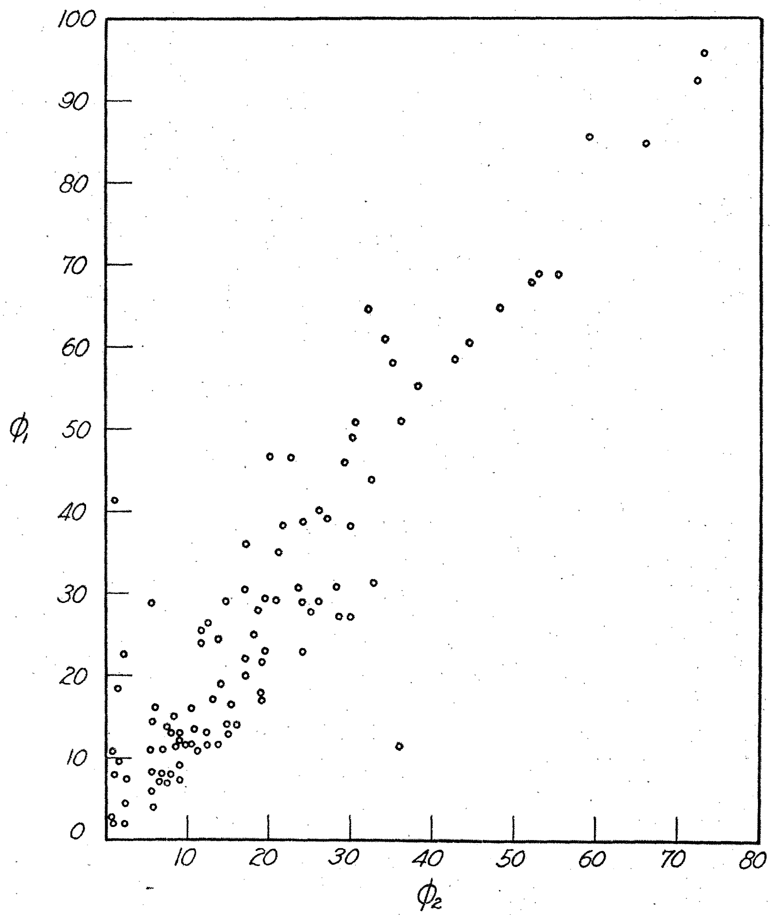

FIGURE 1

Spot diagram showing the correlation between ϕ_1 and ϕ_2. Each dot represents one county.

ing classes: 0–4, 5–9, 10–17, 18–20, 21–44, 45 and over, both figures being inclusive in each case. For each of the 99 counties ϕ was computed on the basis of this grouping of the ages. Call these values ϕ_1. Then for the same 99 counties the population was combined by addition into the 3 age groups 0–9, 10–20 and 21 and over. A new set of ϕ's was then calcu-

lated on the basis of this coarse grouping. Call these values ϕ_2. Then what we wish to know is the value of the correlation $r_{\phi_1\phi_2}$.

This was calculated by the direct product moment method, without grouping and gave as a result

$$r_{\phi_1\phi_2} = + .84 \pm .02.$$

It is at once evident that we are dealing here with a high degree of correlation. That the regression is linear is indicated by the spot diagram of the correlation surface shown in figure 1.

This result indicates that whenever it is impossible, by reason of unsatisfactory tabulation of the original raw statistics, to get any but a coarse age grouping, we may still use the age index ϕ with confidence that it will reflect closely the differences in population in respect of age distribution. The test here given is obviously a severe one, because all adult ages are thrown into one group. But even so, the correlation is high. With such a value of r as this, it is obvious that one could, by means of the appropriate regression equation, determine corrections for the ϕ_2 values, giving predicted or calculated ϕ_1's which would with extreme accuracy approximate the true values.

The means and standard deviations are as follows:

Mean $\phi_1 = 27.27 \pm 1.46$
Standard deviation $\phi_1 = 21.53 \pm 1.03$
Mean $\phi_2 = 19.71 \pm 1.23$
Standard deviation $\phi_2 = 18.14 \pm .87$

As would be expected the ϕ_2 mean is lower than the ϕ_1 mean, and ϕ_2 is less variable, as indicated by the standard deviation, than ϕ_1. The difference in the means is large, but this is of no great importance in the practical use of the age index. It is of much greater practical significance that the difference in the variabilities is not large. If we could trust here the theory of simple sampling (which we probably cannot) the difference between the standard deviations could not be regarded as certainly significant at all. The more important consideration is that ϕ_2 is reflecting in its standard deviation 84.2 per cent of the variation among these 99 Iowa counties in respect of age distribution of their populations, as measured by ϕ_1.

In conclusion it may be stated that since, as is demonstrated in this paper, there is a high correlation between the values of ϕ for extremely coarse age groupings, with the values obtained from finer groupings, this age index may be used with considerable confidence in cases where data on age distribution are available only for a few broad classes covering the life span. All our experience with this age index ϕ since its publication two years ago has confirmed our belief that it is a function of great value to the vital

statistician who wishes to make analytical studies of medical and hygienic data by modern statistical methods.

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THE THERMAL IONIZATION OF GASEOUS ELEMENTS AT HIGH TEMPERATURES

A CONFIRMATION OF THE SAHA THEORY

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In a recent article Eggert[1] has shown that the ordinary thermodynamic expression for the effect of temperature on the equilibrium of chemical reactions, when supplemented by assumptions as to the values of the special constants occurring in it, can be applied to the calculation of the extent to which under different conditions neutral atoms of the gaseous elements are converted into positive ions and free electrons. Saha[2] has shown that the most uncertain factor in Eggert's formulation can be eliminated by introducing the ionization-potential of the element. He has then calculated the dissociation of the first electron from the atoms of many important elements, thus, the extent to which such reactions as $Na = Na^+ + E^-$ take place, where E^- represents electron gas. He has tabulated the values of this percentage ionization at various temperatures and pressures; and he has especially pointed out the great significance which these considerations may have for the interpretation of the spectra of elements under solar and stellar conditions.

These principles are of great interest to astrophysicists, as may be illustrated by the following applications. It has long been known that the solar prominences show strongly the hydrogen and helium lines and the enhanced (H and K) lines of calcium, but not the blue line of calcium nor the familiar lines of sodium or of other alkali elements. The remarkable absence of these lines had not been explained; but the thermodynamic relations now show that the extremely small pressure combined with the high temperature of the prominences must greatly promote the